Example 9: Let $A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$. Calculate $\det(A)$, $\det(B)$, $\det(AB)$, and $\det(A+B)$. Also calculate $\det(A^T)$ and $\det(B^T)$. What do you observe?

$$\det(A) = \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = (0)(3) - (2)(1) = -2$$

$$\det(B) = \begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} = (4)(1) - (2)(3) = -2$$

$$AB = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 14 & 9 \end{bmatrix}$$

$$\det(AB) = \begin{vmatrix} 2 & 1 \\ 14 & 9 \end{vmatrix} = (2)(9) - (14)(1) = 4$$

$$\det(A^T) = \begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix} = -2$$

$$\det(B^T) = \begin{vmatrix} 4 & 2 \\ 3 & 1 \end{vmatrix} = -2$$

$\det(A) = \det(A^T)$ and $\det(B) = \det(B^T)$

$$\det(A) \cdot \det(B) = (-2)(-2) = 4 = \det(AB)$$

*

Question: Is $\det(A+B) = \det(A) + \det(B)$

NO!

$$A+B = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix} \qquad \det(A+B) = \begin{vmatrix} 4 & 4 \\ 4 & 4 \end{vmatrix} = 0$$

$$\det(A) + \det(B) = -2 + (-2) = -4$$

$$\det(A+B) \neq \det(A) + \det(B)$$

> *Theorem 4.8:* If A and B are $n \times n$ matrices, then
>
> $$\det(AB) = \det(A)\det(B)$$
>
> "multiplicative" NOT additive

> *Theorem 4.10:* If A is a $n \times n$ matrix, then
>
> $$\det(A^T) = \det(A)$$